4-3 day 1 The First Derivative Test for Local Extrema

Learning Objectives:

I can use the first derivative test to find local extrema of a function.

I can identify the intervals on which a function is increasing or decreasing.

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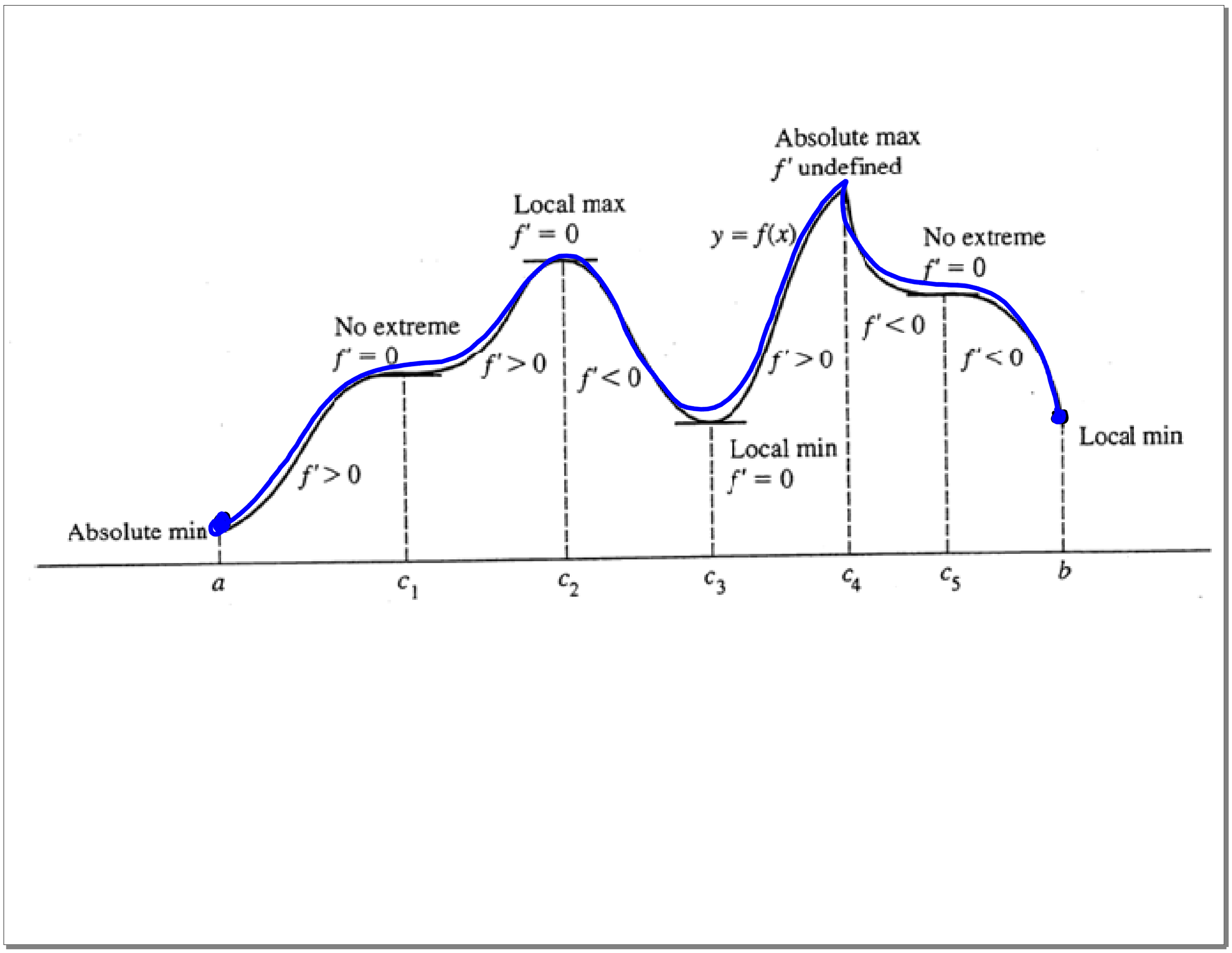
Absolute max
f′ undefined
Local max
f′ = 0
y = f(x)
No extreme
f′ = 0
No extreme
f′ = 0
f′ > 0
f′ < 0
f′ > 0
f′ < 0
f′ < 0
f′ > 0
Local min
f′ = 0
Local min
Absolute min
a
c1
c2
c3
c4
c5
b

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Ex1. Find the critical points of each function. Find the functions local extreme values . Identify the intervals on which the function is increasing/decreasing.

1.) $f(x) = 2x^3 - \frac{11}{2}x^2 - 7x + 5$

$f' = 6x^2 - 11x - 7$

$0 = 6x^2 - 11x - 7$

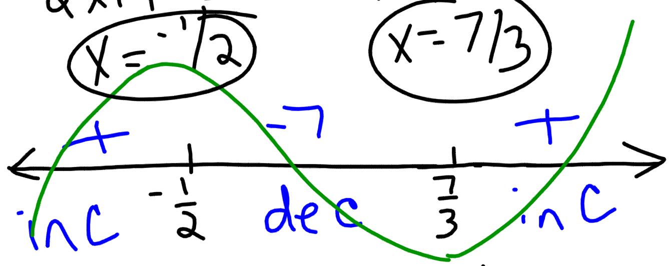
$0 = (2x+1)(3x-7)$

$2x+1=0$ $\quad$ $3x-7=0$

$x = -1/2$ $\quad$ $x = 7/3$

$f'(3) = 6(3)^2 - 11(3) - 7$
$= 54 - 33 - 7$

$f(-1) = 6(-1)^2 - 11(-1) - 7$
$= 6 + 11 - 7$

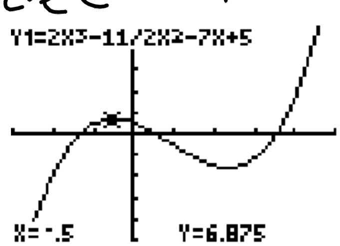


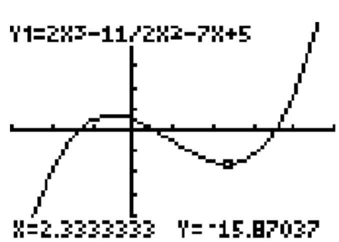
$x = -\frac{1}{2}$ is a max b/c f' changes from + to −.

$x = 7/3$ is a min b/c f' changes from − to +

inc on $(-\infty, -\frac{1}{2}]$ & $[\frac{7}{3}, \infty)$ b/c f' is +

dec on $[-\frac{1}{2}, \frac{1}{3}]$ b/c $f' = -$.





2.) $y = \underbrace{(x^2 - 3)}_{f}\underbrace{e^x}_{g}$

② $y = (x^2-3)e^x$

$y' = (2x)e^x + (x^2-3)e^x = 2xe^x + x^2e^x - 3e^x$

$0 = 2xe^x + x^2e^x - 3e^x$

$0 = 2x + x^2 - 3$

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$x = -3$

$x = 1$

$-8e^{-4} + 16e^{-4} - 3e^{-4}$

$4e^{2} + 4e^{2} - 3e^{2}$

$+ \quad - \quad +$

$-4 \quad -3 \quad 0 \quad 1 \quad 2$

$e^x(x^2+2x-3)$

$0 = e^x(x+3)(x-1)$

$e^x = 0$ N.S. $\qquad x+3=0$, $x=-3$ $\qquad x-1=0$, $x=1$

$x=-3$ is a max b/c f' changes from "+" to "–"

$x=1$ is a min b/c f' changes from "+" to "–" (− +)

inc on $(-\infty,-3) + (1,\infty)$ b/c f' is "+"

dec on $(-3,1)$ b/c f' is "–"

3.) $y = x^4 - 2x^3$

(3.) $f(x) = x^4 - 2x^3$

$f'(x) = 4x^3 - 6x^2$

$0 = 4x^3 - 6x^2$

$0 = 2x^2(2x - 3)$

$2x^2 = 0 \qquad 2x - 3 = 0$

$x = 0 \qquad x = 3/2$

The graph suggests that there is 1 critical pt.

—   —   +

dec   0   dec $1\frac{1}{2}$ inc

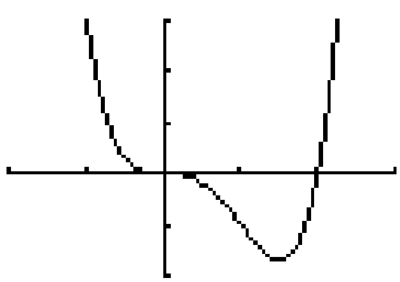
interval	$x < 0$	$0 < x < 1\frac{1}{2}$	$x > 1\frac{1}{2}$
Sign of f'	—	—	+
behavior of f	decreasing	decreasing	increasing

at $x = 0$, since the fcn is decreasing on both sides of $x = 0$, $x = 0$ is <u>NOT</u> an extrema

at $x = 3/2$, the fcn is decreasing on the left side but increasing on the rt side, so $x = 3/2$ must be a min.

increasing $[3/2, \infty)$ b/c $f' = +$

decreasing $(-\infty, 0]$ & $[0, 3/2]$ b/c $f' = -$



4.) $g(x)=x^{2/3}+x$

(4.) $y = x^{2/3} + x$

$y' = \frac{2}{3}x^{-1/3} + 1 = 0$

$\frac{-1}{2/3} = \frac{\cancel{2/3}\, x^{-1/3}}{\cancel{2/3}}$

$\left(\frac{-3}{2}\right)^{-3} = \left(x^{-1/3}\right)^{-3}$

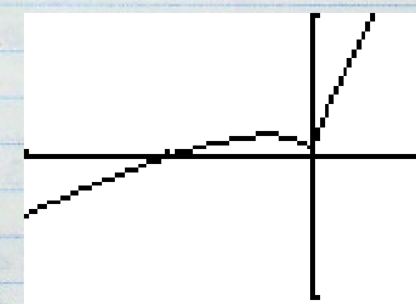
$x = \frac{-8}{27}$

$y' = \frac{2}{3}x^{-1/3}$

$y' = \frac{2}{3}\,\frac{1}{\sqrt[3]{x}} + 1$

$y' = \frac{2}{3\sqrt[3]{x}} + 1$

y' is undef @ $x=0$



$+$ $-$ $+$

$-\frac{8}{27}$ 0 inc

$x=0$ is a min

$x = -\frac{8}{27}$ is a max

OR $\frac{-3}{2} = \frac{1}{\sqrt[3]{x}}$

$\frac{-3\sqrt[3]{x}}{-3} = \frac{2}{-3}$

$\left(\sqrt[3]{x}\right)^3 = \left(-\frac{2}{3}\right)^3$

$x = \frac{-8}{27}$

First Derivative Test for Local Extrema

$f'(x)>0$ f(x) is increasing
$f'(x)<0$ f(x) is decreasing

1st derivatives find slope (tell us if the function is increasing or decreasing) and are used to find extrema (max's and min's).

A sign change in the first derivative indicates that the function has changed from increasing to decreasing or vice versa. You must observe a sign change to be sure that an extrema is present.

Homework

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